Abstract of paper [1].

Let \( k \geq 2 \) be a fixed integer and \( p \) denote a prime. For any \( n \in \mathbb{N} \) such that the polynomial \( x^k - n \) is irreducible over \( \mathbb{Q} \) let

\[
R_k(n) := \sum_{h+m^k=n} \Lambda(h), \quad \rho_k(n,p) := |\{m \mod p : m^k \equiv n \mod p\}|
\]

and

\[
\mathcal{S}_k(n) := \prod_p \left(1 - \frac{\rho_k(n,p) - 1}{p-1}\right) = \prod_p \left(1 - \frac{\rho_k(n,p)}{p}\right) / \left(1 - \frac{1}{p}\right).
\]

One expects that for the integers \( n \) we are considering one has \( R_k(n) \sim n^{1/k} \mathcal{S}_k(n) \) as \( n \to \infty \). Let \( L := \log N \),

\[
R_k^*(n) := \sum_{h+m^k=n \atop N-Y \leq h \leq N \atop Y/2 \leq m^k \leq 3/2Y} \Lambda(h), \quad \text{and} \quad P_k^*(n) := \frac{1}{k} \sum_{h+m=n \atop N-Y \leq h \leq N \atop Y/2 \leq m \leq 3/2Y} m^{1/k-1}.
\]

Building on previous work by Perelli & Pintz in the case \( k = 2 \), we prove

**Theorem 1** Let \( k \geq 3 \), \( \varepsilon \), \( A > 0 \), \( N^{7/12+\varepsilon} \leq Y \leq N \) and \( \max(Y^{1-1/k+\varepsilon}, N^{1/2+\varepsilon}) \leq H \leq Y \). Then

\[
\sum_{N \leq n \leq N+H} |R_k^*(n) - P_k^*(n)\mathcal{S}_k(n)|^2 \ll_{\varepsilon,A,k} HY^{2/k}L^{-A},
\]

where the \( \ast \) means that the sum is over \( n \in \mathbb{N} \) such that \( x^k - n \) is irreducible over \( \mathbb{Q} \).

**Theorem 2** Assume the Generalized Riemann Hypothesis and let \( k \geq 2 \), \( \varepsilon > 0 \). Then

\[
E_k(N) := |\{N \leq n \leq 2N : n \neq p+m^k\}| \ll_{k,\varepsilon} N^{1+\varepsilon-2/(kK)}
\]

where \( K := 2^{k-1} \).

Theorem 2 is obtained by means of Hardy & Littlewood’s circle method, and Weyl’s inequality, by a suitable treatment of the singular series.

**References**