Abstract of paper [1].

Let \( k \geq 2 \) be an integer, and set \( E_k(X) := | \{ n \leq X : n \neq m^k, n \text{ is not a sum of a prime and a } k\text{-th power} \} | \). We prove that there exists \( \delta = \delta(k) > 0 \) such that \( E_k(X) \ll_k X^{1-\delta} \), by means of a suitable application of the circle method, essentially a variant of Montgomery & Vaughan's method (Acta Arithmetica 1975). The proof is similar to the one given by Brünner, Perelli & Pintz (Acta Math. Hungarica 1989) in the case \( k = 2 \), the main new difficulty being in the treatment of the singular series.

References